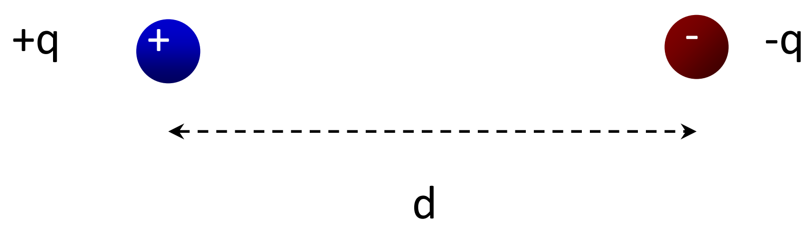
Lecture-7

Electric field in material space
Polarization in dielectrics, dielectric constants,

1. A Dipole

A combination of two electric charges with equal magnitude and opposite sign is called a dipole.

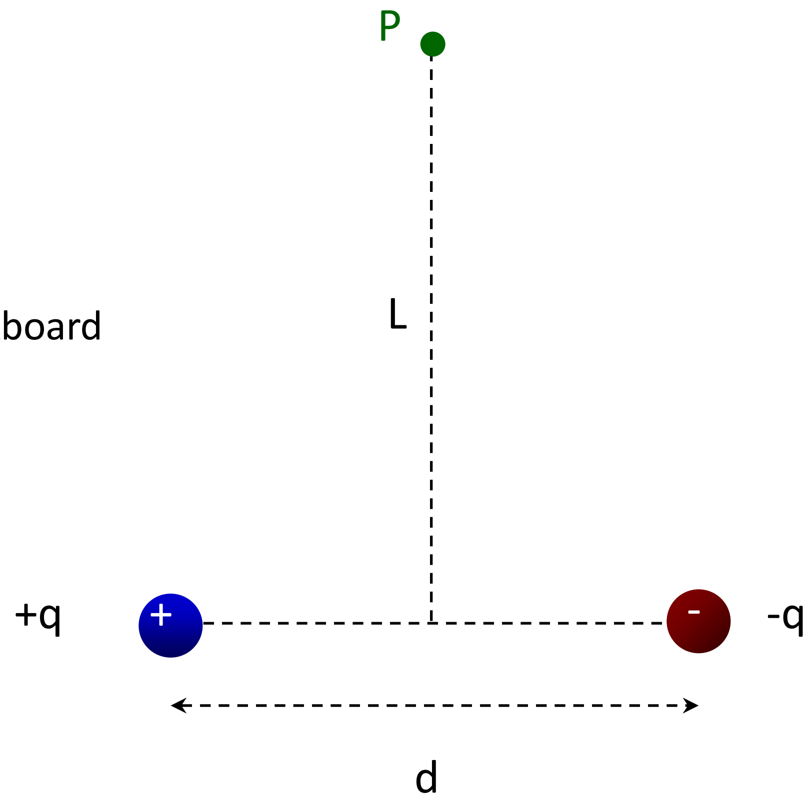


The charge on this dipole is q (not zero, not +q, not –q, not 2q). The distance between the charges is d. Dipoles are “everywhere” in nature.

Dipole

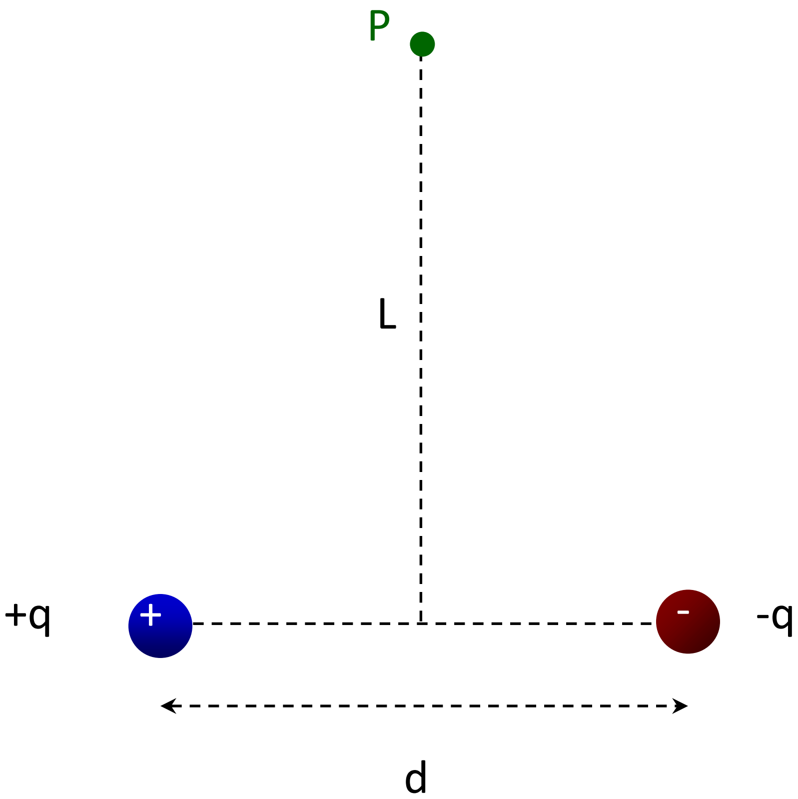
Example: calculate the electric field at point P, which lies on the perpendicular bisector a distance L from a dipole of charge q.

to be worked at the blackboard



$$\vec{E} = k \sum_i \hat{r} \frac{q_i}{r_i^2}$$

$$E = \frac{qd}{4\pi\varepsilon_o r^3}$$



$$E = \frac{qd}{4\pi\varepsilon_o r^3}$$

Electric Dipole in an External Electric Field

An electric dipole consists of two charges +q and -q, equal in magnitude but opposite in sign, separated by a fixed distance d. q is the "charge on the dipole."

Earlier, I calculated the electric field along the perpendicular bisector of a dipole (this equation gives the magnitude only).

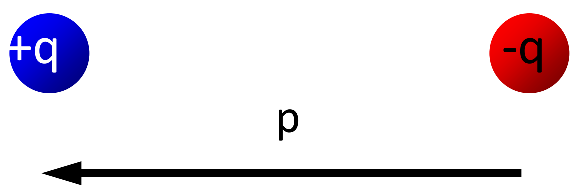
$$E = \frac{qd}{4\pi\varepsilon_o r^3}.$$

The electric field depends on the product qd.

q and d are parameters that specify the dipole; we define the "dipole moment" of a dipole to be the vector

$$\vec{p} = q\vec{d},$$

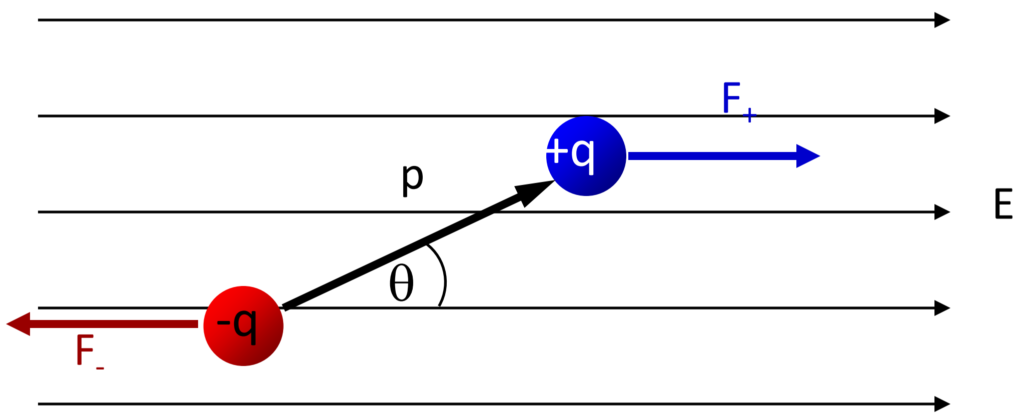
where the direction of p is from negative to positive (NOT away from +).



To help you remember the direction of p, this is on the equation sheet:

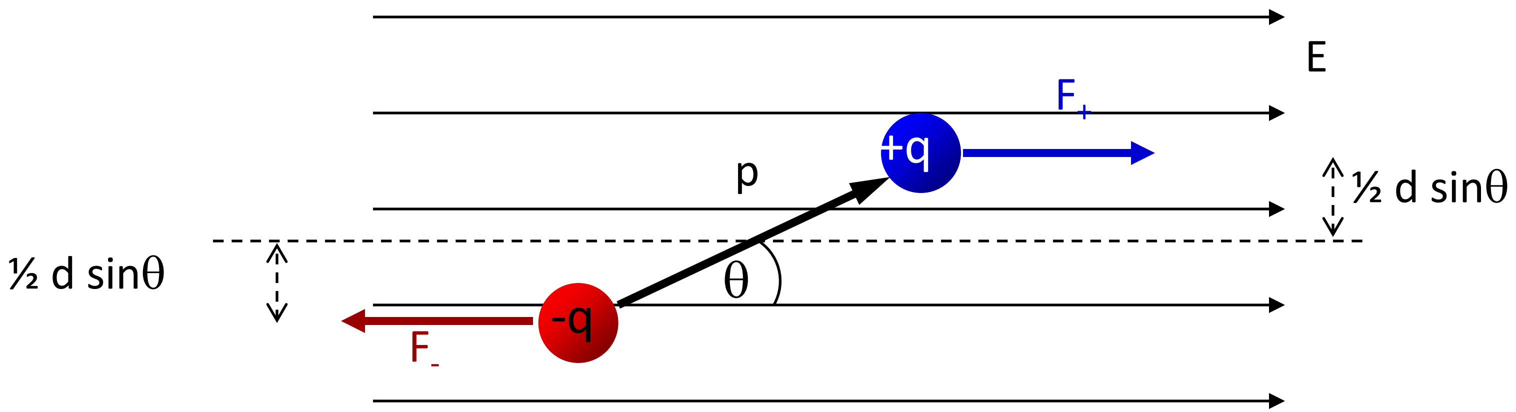
$$\boxed{\vec{p} = |q|\,d,\ \text{from} - \text{to plus}}$$

A dipole in a uniform electric field experiences no net force, but probably experiences a torque…



There is no net force on the dipole:

$$\sum \vec{F} = \vec{F}_- + \vec{F}_+ = -q\vec{E} + q\vec{E} = 0.$$

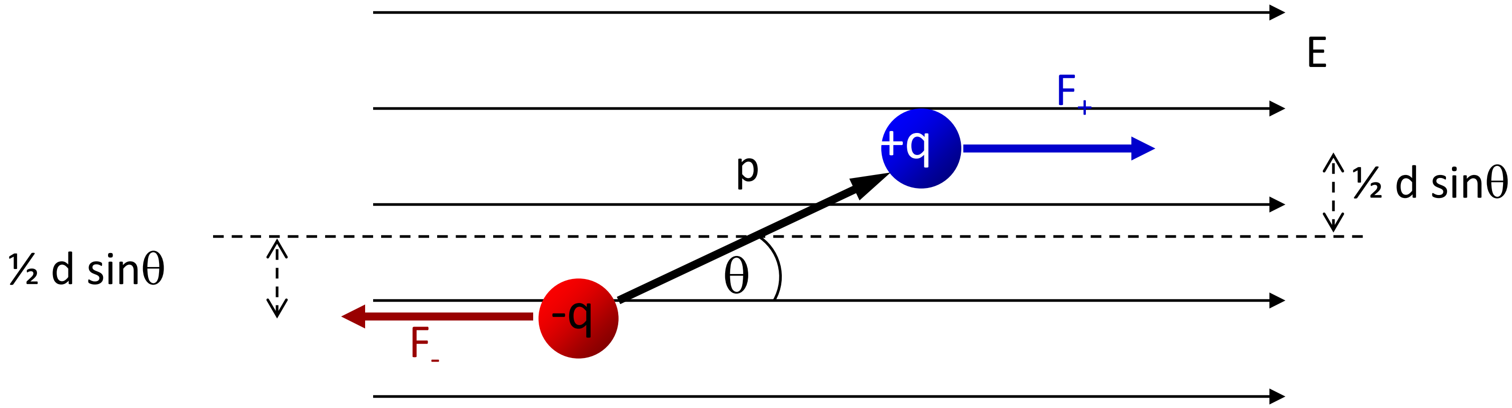


If we choose the midpoint of the dipole as the origin for calculating the torque, we find

$$\sum \tau = \tau_+ + \tau_- = \frac{d\sin\theta}{2} qE + \frac{d\sin\theta}{2} qE = qdE\sin\theta,$$

and in this case the direction is into the plane of the figure. Expressed as a vector,

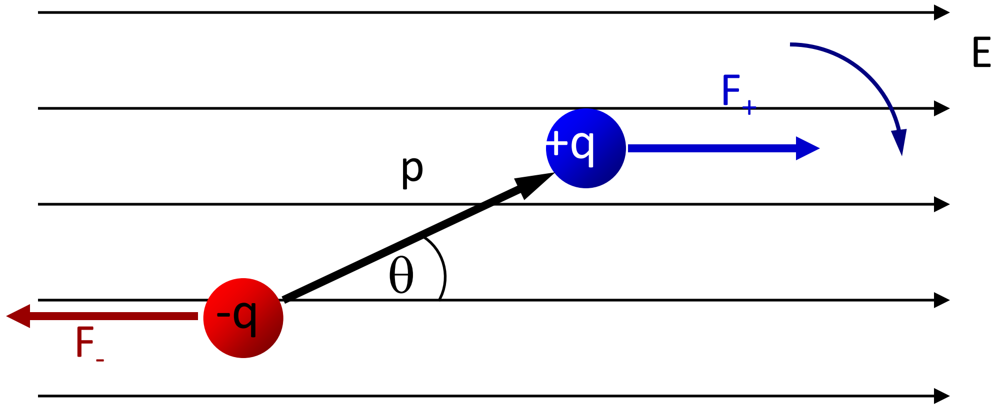
$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}.}$$



The torque's magnitude is $p\,E\sin\theta$ and the direction is given by the right-hand rule.

What is the maximum torque?

Energy of an Electric Dipole in an External Electric Field



If the dipole is free to rotate, the electric field does work* to rotate the dipole.

$$W = -pE(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}).$$

The work depends only on the initial and final coordinates, and not on how you go from initial to final.

If a force is conservative, you can define a potential energy associated with it.

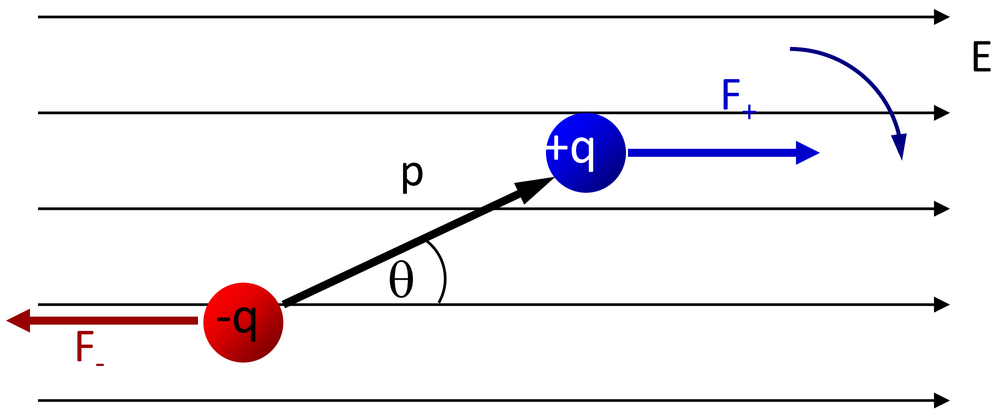
Because the electric force is conservative, we can define a potential energy for a dipole. The equation for work

$$W = -pE(\cos\theta_{\mathrm{initial}} - \cos\theta_{\mathrm{final}})$$

suggests we should define

$$U_{\mathrm{dipole}} = -pE\cos\theta.$$

$$U_{dipole} = -pE\cos\theta$$



With the definition on the previous slide, U is zero when $\theta=\pi/2$.

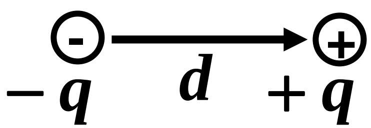
U is maximum when $\cos\theta=-1$, or $\theta=\pi$ (a point of unstable equilibrium).
U is minimum when $\cos\theta=+1$, or $\theta=0$ (stable equilibrium).
It is “better” to express the dipole potential energy as

$$\boxed{U_{dipole} = -\vec{p}\cdot\vec{E}.}$$

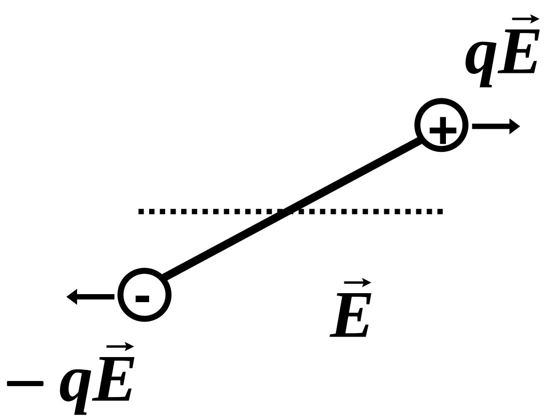
Dielectrics

In dielectrics, there are no free charges, but bound charges, i.e. electric dipoles are present.

Definition



dipole moment $\vec{p} = q\vec{d}$ **(- $\rightarrow$ +)**



Torque felt

$$\vec{\tau} = \vec{p} \times \vec{E}$$

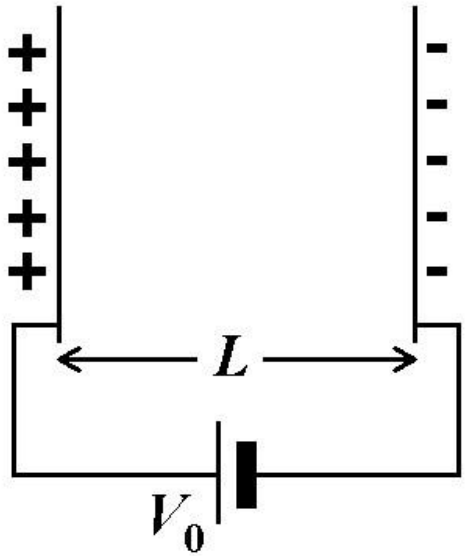
Potential energy

$$V = -\vec{p} \cdot \vec{E}$$

In discussing dielectric materials

Polarization $\vec{P}$ **= # of dipole moment / unit volume**

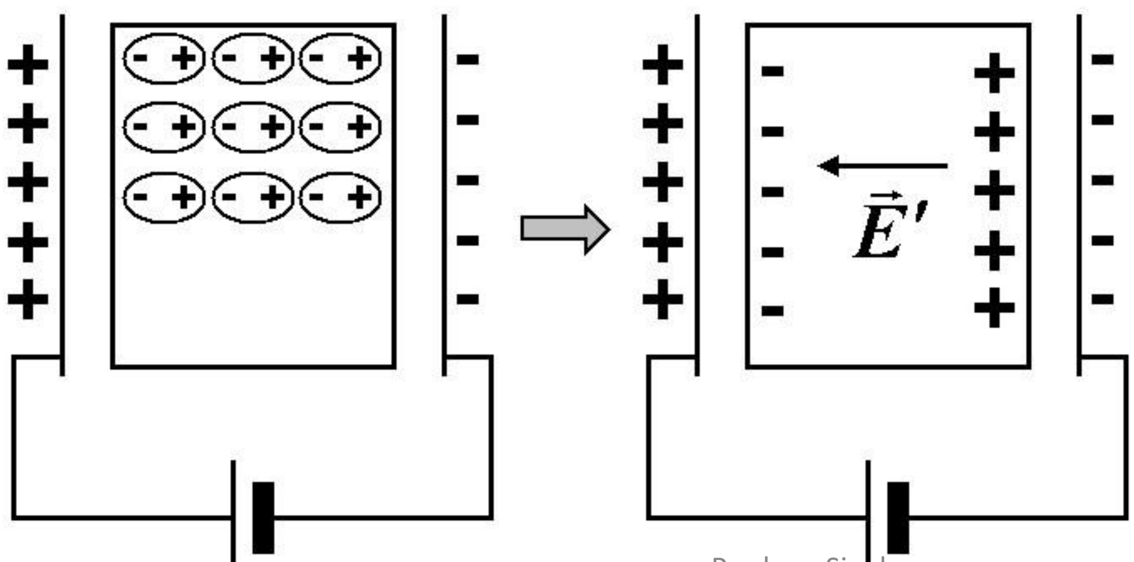
= $N\vec{P}$



Electric field inside the parallel plate :

$$\vec{E}_0 = \frac{V_0}{L}$$

Now, insert a slab of dielectric $\Rightarrow$ modify the field to a new value $\vec{E}$



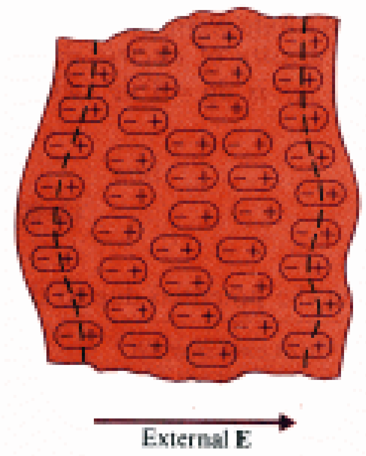
2. Dielectrics in Static Electric Field

The presence of an external electric field causes a force to be exerted on each charged particle and results in small displacements of positive and negative charges in opposite directions. These displacements polarize a dielectric material and create electric dipoles, which equivalent to an volume charge density ρ_p .

The polarized charge density is related to the polarization vector **P** by

$$\rho_p = -\nabla \cdot \bar{P}$$

The "-" sign means that the field generated by the electric dipoles is in the opposite direction of external electric field *E*.



A polarized dielectric medium

The divergence postulation must be modified to include the effect of ρ_p

$$\nabla \cdot \bar{E} = \frac{1}{\varepsilon_0}(\rho + \rho_p)$$

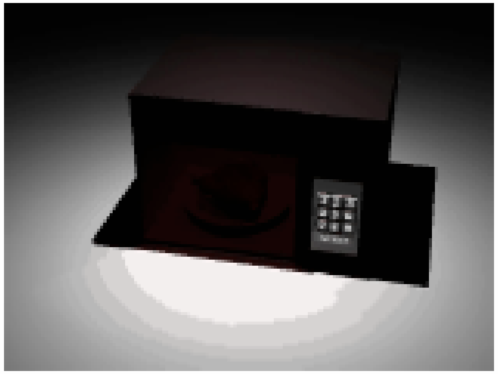
$$\boxed{\rho_p = -\nabla \cdot \bar{P}}$$

or $$\nabla \cdot (\varepsilon_0 \bar{E} + \bar{P}) = \rho$$

We now define a new fundamental field quantity, the ***electric flux density***, or ***electric displacement***, **D**, such that

- $$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} \quad (C/m^2)$$

or $$\nabla \cdot \bar{D} = \rho \quad (C/m^3)$$



3. Electric Flux Density and Dielectric Constant

When the dielectric properties of the medium are linear and isotropic, the polarization is directly proportional to the electric field intensity, and the proportionality constant is independent of the direction of the field, We have

$$\bar{P} = \varepsilon_0 \chi_e \bar{E}$$

where χ_e is a dimensionless quantity called electric susceptibility. It is obvious that

- $\bar{D} = \varepsilon_0 (1 + \chi_e) \bar{E} = \varepsilon_0 \varepsilon_r \bar{E} = \varepsilon \; \bar{E} \;\; (C/m^2)$

Relative permittivity or dielectric constant

Absolute permittivity

Material	Relative Permittivity, ϵ_r
Air	1.0
Bakelite	5.0
Glass	4–10
Mica	6.0
Oil	2.3
Paper	2–4
Parafin wax	2.2
Plexiglass	3.4
Polyethylene	2.3
Polystyrene	2.6
Porcelain	5.7
Rubber	2.3–4.0
Soil (dry)	3–4
Teflon	2.1
Water (distilled)	80
Seawater	72

$$\nabla \bullet \overline{D} = \rho \left(\frac{\mathrm{C}}{\mathrm{m}^3} \right) \qquad \nabla \times \overline{E} = 0$$

$$\int_v \nabla \bullet D dv = \int_v \rho dv \quad \Leftrightarrow \quad \oint_s D \bullet ds = Q \quad (C)$$

Gauss's Law: The total outward flux of the dielectric displacement (or simply the outward flux) over any closed surface is equal to the total free charge enclosed in the surface

$$\overline{D} = \varepsilon_0 \overline{E} + \overline{P} = \varepsilon_0 \overline{E} + \varepsilon_0 \chi_e \overline{E} =$$

$$= \varepsilon_0 (1 + \chi_e) \overline{E} = \varepsilon_0 \varepsilon_r \overline{\mathrm{E}} = \varepsilon \overline{E} \quad \left(\frac{\mathrm{C}}{\mathrm{m}^2} \right)$$

Where- ε is the absolute permittivity (F/m)

-ε_r is the relative permittivity or the dielectric constant of the medium

-ε_0 is the permittivity of free space

-χ_e is the electric susceptibility (dimensionless)